

# Vortices and type-I superconductivity in neutron stars

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In a recent paper by Link, it was pointed out that the standard picture of the neutron star core, composed of a mixture of a neutron superfluid and a proton type-II superconductor, is inconsistent with observations of long period precession in isolated pulsars. In the following we will show that intervortex force between the magnetic flux tubes may be attractive resulting in a type-I (rather than type-II) superconductor. In this case the magnetic field cannot exist in the form of magnetic flux tubes, supporting Link's observation. This behavior of the system is due to the strong interaction between the proton-neutron Cooper pairs, which was previously ignored. We also calculate the critical magnetic fields  $H_c$  and  $H_{c2}$  for type-I/II superconductors. These results also support our claim of type-I superconductivity in the cores of neutron stars.

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## I. INTRODUCTION

Recently, it was pointed out by Link [1] that observations of long period precession [2] may be inconsistent with the standard picture of the interior of a neutron star. In the conventional picture, the extremely dense interior is mainly composed of neutrons, with a small amount of protons and electrons in beta equilibrium. The neutrons form  $^3P_2$  Cooper pairs and Bose condense to a superfluid state, while the protons form  $^1S_0$  Cooper pairs and Bose condense to give a superconductor as well (see e.g. Ref. [3] for a recent review). It is generally assumed that the proton superconductor is a type-II superconductor, which means that it supports a stable lattice of magnetic flux tubes in the presence of a magnetic field. This belief is based on simple estimations of the coherence length and the London penetration depth which ambiguously imply a type-II superconductivity. In addition, the rotation of a neutron star causes a lattice of quantized vortices to form in the superfluid neutron state, similar to the observed vortices that form when superfluid  $^4\text{He}$  is rotated fast enough. The axis of rotation and the axis the magnetic field are not aligned. This, coupled with the fact that the two different types of vortices interact quite strongly lead to the suggestion that the observed long period precession may require reexamining the picture of type-II superconductivity inside neutron stars [1] that follows from the standard analysis when only a single proton field is considered. A possible resolution of this "apparent contradiction" was suggested in our recent letter [4], where we argued that if one takes into account that the Cooper pairs of neutrons are also present in the system and they strongly interact with proton Cooper pairs, the superconductor may in fact be type-I, even when a naive analysis that only includes the proton degrees of freedom seems to indicate type-II behavior. If this scenario is realized in nature, this means that the interior of neutron stars would exhibit the Meissner effect and therefore would not support a stable lattice of

magnetic flux tubes. This would resolve the apparent discrepancy [1] between the observation of long period precession [2] and the typical parameters of the neutron stars which naively suggest type-II superconductivity in neutron stars. The main goal of this paper is to provide a detailed analysis with complete calculations supporting the claim of our short letter [4].

It is well known that type-II superconductors have magnetic flux tubes in the presence of a magnetic field. In the interior of neutron stars, which is a mixture of neutron and proton superfluids, the proton superfluid will form a vortex lattice of magnetic flux tubes if the superconductor is type-II. Inside the core of these vortices, the proton condensate vanishes, and the core is filled with normal protons resulting in the restoration of the broken  $U(1)_{EM}$  symmetry. For accepted estimates of proton correlation length and London penetration depth, the distant proton vortices repel each other leading to formation of a stable vortex lattice. This is the standard picture realized in conventional type-II superconductors. However, there are many situations where this picture will be qualitatively modified. For example, if there is a second component (such as a neutron component in our specific case), it may be energetically favorable for the cores of vortices to be filled with a nonzero condensate of this second component, as it was originally suggested in the cosmological context by Witten [5, 6]. There are numerous examples of physical systems where this phenomena occurs: superconducting cosmic strings in cosmology, magnetic flux tubes in the high  $T_c$  superconductors, Bose-Einstein condensates, superfluid  $^3\text{He}$ , and high baryon density quark matter [5, 6, 7, 8, 9, 10, 11]. Given this, one might guess that such nontrivial vortex structure may occur in the core of neutron stars where we have another example of a two component system. We shall argue in what follows that if the interaction between proton and neutron Cooper pairs is approximately equal (a precise condition of this "approximately" will be derived below), the vortex-vortex interaction will be

modified and the system will be a type-I superconductor where the magnetic field is completely expelled from the bulk.<sup>1</sup> The main assumption that we are making is that the interactions between the proton and neutron Cooper pairs are approximately equal, leading to an approximate  $U(2)$  symmetry. We believe that this assumption is justified by the original isospin  $SU(2)$  symmetry of the neutrons and protons. The result of this is that the proton vortices or magnetic flux tubes have nontrivial core structure. The superfluid density of the neutrons is larger in the vortex core than at spatial infinity. In addition, the size of the vortex core and the asymptotic behavior of the proton condensate is modified due to the additional neutron condensate. The most important result of these effects is that the interaction between distant proton vortices may be attractive in a physically realizable region of parameter space leading to type-I behavior: destruction of the proton vortex lattice and expulsion of the magnetic flux from the superconducting region of the neutron star.

This paper is organized as follows. In Sec. II we will introduce the free energy that describes a two component (neutron-proton) superfluid/superconducting system and show that the addition of the second component (neutron) leads to nontrivial core structure, which alters the properties of the magnetic flux tube. In Sec. III, we will give two different calculations of the interaction between two widely separated vortices. These two different methods lead to the same conclusion, that the interior of a neutron star may be a type-I superconductor. In the subsequent section, Sec. IV, we will calculate the critical magnetic fields  $H_c$  and  $H_{c2}$  (the magnetic fields above which superconductivity is destroyed in type-I and type-II superconductors, respectively). These results confirm our findings in Sec. III that type-I superconductivity may occur in the interior of neutron stars. In Sec. V we will end with concluding remarks on possible implications of our results. Specifically, we will comment on the possible nature of glitches (observed in many systems) when the environment of the neutron star core is a type-I superconductor.

## II. STRUCTURE OF MAGNETIC FLUX TUBES

We start by considering the following effective Landau-Ginsburg free energy that describes a two component superfluid Bose condensed system. In our system, we have a proton condensate described by  $\psi_1$  and a neutron condensate described by  $\psi_2$ . We do not consider the normal component of the protons and neutrons with their specific excitations, only the superfluid component. The  $\psi_1$

field with electric charge  $q$  (which is actually twice the fundamental charge of the proton,  $q = 2|e|$ ) interacts with the gauge field  $\mathbf{A}$ , with  $\mathbf{B} = \nabla \times \mathbf{A}$ . The two dimensional free energy reads (we neglect the dependence on third direction along the vortex such that  $\mathcal{F}$  measures the free energy per unit length):

$$\mathcal{F} = \int d^2r \left( \frac{\hbar^2}{2m_c} (|\nabla - \frac{iq}{\hbar c} \mathbf{A}| \psi_1|^2 + |\nabla \psi_2|^2) + \frac{\mathbf{B}^2}{8\pi} + V(|\psi_1|^2, |\psi_2|^2) \right) \quad (1)$$

where  $m_c = 2m$  and  $m$  is the mass of the nucleon. Here we have moved the effective mass difference of the proton and neutron Cooper pairs onto the interaction potential  $V$ . In the free energy given above, we have ignored the term coupling the proton and neutron superfluid velocities, which gives rise to the Andreev-Bashkin effect [12], as it is not important in our discussion. We have also ignored the fact that the neutron condensate has a non-trivial  $^3P_2$  order parameter as only the magnitude of the neutron condensate is relevant to the effect described below.

The free energy (1) is invariant under a  $U(1)_1 \times U(1)_2$  symmetry associated with respective phase rotations of fields  $\psi_1$  and  $\psi_2$ , which corresponds to the conservation of the number of Cooper pairs for each species of particles. Moreover, we assume, that the proton and neutron Cooper pairs interact approximately in the same way. Therefore, the interaction potential  $V$  can be approximately written as  $V(|\psi_1|^2, |\psi_2|^2) \approx U(|\psi_1|^2 + |\psi_2|^2)$ .

Naively, one might think that such an assumption can not be justified due to the well-known differences in structure of the Fermi-surfaces and gaps (which strongly depend on the Fermi energies) of neutron and proton Cooper pairs in neutron stars. However, this does not imply that the interaction at large distances (much larger than the inverse gap) between proton and neutron Cooper pairs (not between protons and neutrons) is very different or that their respective Bose chemical potentials  $\mu_i$  (do not confuse with original Fermi chemical potentials corresponding to protons and neutrons) are very different. We expect that at very large distances which are relevant for the analysis of the vortex-vortex interaction, the internal structure of the gap as well as the differences in the densities of proton and neutron Cooper pairs  $n_{1,2}$  (do not confuse with proton and neutron densities) are not very important provided that scattering lengths of different Cooper pair species are approximately the same. In the case where  $m_d = m_u$  and electromagnetic interactions are neglected, we expect that the scattering lengths for proton and neutron Cooper pairs would be exactly the same. When  $m_d \neq m_u$  we expect that the effect of the asymmetry due to the renormalization with the environment would be proportional to  $m_d - m_u$ . In principle, some mild singularity such as  $\ln(m_d - m_u)$  may occur due to the renormalization. However, we do not expect that a strong singularity (such as  $(m_d - m_u)^{-1}$ ) that is capable of eliminating the original small factor  $(m_d - m_u)$

<sup>1</sup> In reality, the magnetic field must be present in the neutron star interior. The way this picture may be realized in nature is through the formation of domains of superconductor matter and normal matter.

will appear. In other words, if interaction of a Cooper pair at  $k \rightarrow 0$  is symmetric for protons and neutrons, it is not essential that the system itself is an asymmetric one, with  $n_1 \neq n_2$ . Such an asymmetry can be easily adjusted by slightly different Bose chemical potentials, see below, such that the effective Lagrangian remains symmetric. Therefore, when the potential is expressed in terms of Bose chemical potentials (macrocanonical description) rather than in terms of densities, we expect the potential retains its original symmetries.

Therefore, we believe that our approximate  $U(2)$  symmetry is somewhat justified by the original isospin symmetry of the original protons and neutrons, however this symmetry is not exactly equivalent to the conventional isotopical  $SU(2)$  symmetry. In reality this  $U(2)$  symmetry is explicitly slightly broken, and the potential  $V$  has a minimum at  $|\psi_1|^2 = n_1, |\psi_2|^2 = n_2$ , where the bulk proton and neutron superfluid densities  $n_1$  and  $n_2$  are both non-zero. Hence in the ground state,  $\langle |\psi_i|^2 \rangle = n_i$ ,  $i = 1, 2$ , and both  $U(1)$  symmetries are spontaneously broken. In our letter [4], we presented some general arguments for a generic potential  $V(|\psi_1|^2, |\psi_2|^2)$  which has the symmetry properties discussed above. In order to be more specific and give the details of our calculations, in this work we will use the following standard  $\phi^4$ -type potential:

$$V = -\mu_1 |\psi_1|^2 - \mu_2 |\psi_2|^2 + \frac{a_{11}}{2} |\psi_1|^4 + \frac{a_{22}}{2} |\psi_2|^4 + a_{12} |\psi_1|^2 |\psi_2|^2 \quad (2)$$

where  $\mu_i$  is the chemical potential of the  $i^{\text{th}}$  component and  $a_{ij}$  is proportional to the scattering length  $l_{ij}$  between the  $i^{\text{th}}$  and  $j^{\text{th}}$  components,  $a_{ij} = 4\pi\hbar^2 l_{ij}/m_c$ . In our nonrelativistic formalism, the fields  $\psi_i$  have energy dimension 3/2 and therefore the particle density is  $n_i = \langle |\psi_i|^2 \rangle$ ,  $i = 1, 2$ .

Let's analyze the vacuum structure of our potential (2). In the limit of exact  $U(2)$  symmetry  $\mu_i = \mu$ ,  $a_{ij} = a$ , the vacuum manifold is given by the three sphere  $|\psi_1|^2 + |\psi_2|^2 = \mu/a$ . We are, however, interested in the case when the  $U(2)$  symmetry is explicitly broken to  $U(1) \times U(1)$ , giving a particular pattern of proton and neutron condensation  $\langle |\psi_i|^2 \rangle = n_i$ . It is natural that even very small  $U(2)$  symmetry violating terms are capable of selecting a particular vacuum on the original degenerate manifold. First, consider the case when the fourth order couplings are fully degenerate  $a_{ij} = a$ , while the chemical potentials are slightly different  $\mu_1 = \mu - \delta\mu$ ,  $\mu_2 = \mu + \delta\mu$ . In this case, the condensation pattern is determined solely by the sign of  $\delta\mu$ . If  $\delta\mu > 0$  then neutrons condense,  $\langle |\psi_2|^2 \rangle = \mu_2/a_{22}$ , while protons remain uncondensed,  $\langle \psi_1 \rangle = 0$ ; if  $\delta\mu < 0$  then protons condense,  $\langle |\psi_1|^2 \rangle = \mu_1/a_{11}$ , while neutrons remain uncondensed,  $\langle \psi_2 \rangle = 0$ . Observe, that a very small  $U(2)$  violating change in chemical potentials  $\mu_1, \mu_2$  produces a very large asymmetry of proton and neutron Cooper pair densities  $n_1, n_2$ .

Now consider a more general situation in which all chemical potentials  $\mu_1, \mu_2$ , and fourth order couplings  $a_{11}, a_{22}, a_{12}$  are non-degenerate. In this case, a new phase is possible, where both proton and neutron condensates appear. In this phase, the proton and neutron Cooper pair densities are given by:

$$n_1 = \frac{a_{22}\mu_1 - a_{12}\mu_2}{a_{11}a_{22} - a_{12}^2}, \quad (3)$$

$$n_2 = \frac{a_{11}\mu_2 - a_{12}\mu_1}{a_{11}a_{22} - a_{12}^2}. \quad (4)$$

This particular vacuum will be realized if and only if,

$$a_{22}\mu_1 - a_{12}\mu_2 > 0, \quad (5)$$

$$a_{11}\mu_2 - a_{12}\mu_1 > 0. \quad (6)$$

Notice that conditions (5,6) imply  $a_{11}a_{22} - a_{12}^2 > 0$ . If conditions (5,6) are not satisfied then only one condensate will appear, similar to the case of degenerate  $a_{ij}$ 's already described above: if  $\mu_1^2/a_{11} < \mu_2^2/a_{22}$  then  $n_2 = \mu_2/a_{22}$ ,  $n_1 = 0$ ; if  $\mu_1^2/a_{11} > \mu_2^2/a_{22}$  then  $n_1 = \mu_1/a_{11}$ ,  $n_2 = 0$ .

Throughout the rest of the paper, we will be working in the sector where both  $\psi_1$  and  $\psi_2$  obtain non-zero expectation values, as this is the situation, which is believed to be realized in neutron star interiors. In this case, the chemical potentials  $\mu_1, \mu_2$  are fixed by the equilibrium Cooper pair densities  $n_1, n_2$  through equations (3,4). It is convenient to assume the following particular parametrization of explicit  $U(2)$  violation:  $\mu_1 = \mu - \delta\mu$ ,  $\mu_2 = \mu + \delta\mu$ , where  $\delta\mu/\mu \ll 1$ , and  $a_{11} = a_{22} = a$ ,  $a_{12} = a - \delta a$ , where  $\delta a/a \ll 1$ . In this case the equations for Cooper pair densities (3,4) reduce to:

$$n_1 = \frac{\mu}{2a - \delta a} - \frac{\delta\mu}{\delta a} \approx \frac{\mu}{2a} - \frac{\delta\mu}{\delta a}, \quad (7)$$

$$n_2 = \frac{\mu}{2a - \delta a} + \frac{\delta\mu}{\delta a} \approx \frac{\mu}{2a} + \frac{\delta\mu}{\delta a} \quad (8)$$

and conditions for stability of the vacuum (5,6) reduce to

$$\frac{|\delta\mu|}{\mu} < \frac{\delta a}{2a - \delta a} \approx \frac{\delta a}{2a} \quad (9)$$

The approximation made in the second part of equations (7,8,9) neglects terms of order  $\delta a/a$ , which are small in the limit of approximate  $U(2)$  symmetry. An important quantity for the analysis that follows will be the ratio of proton Cooper pair density to neutron Cooper pair density,  $\gamma \equiv n_1/n_2$ . A typical value of  $\gamma$  in the core of a neutron star is expected to be quite small, 5 – 15%. Thus, we will often use the limit  $\gamma \ll 1$  in our discussion. However, one should remark here that our qualitative results do not depend on the value of  $\gamma$ , as we will explain in what follows. As already mentioned, the strong deviation of  $\gamma$  from 1 does not imply a large asymmetry in the interaction between different species of particles. As is clear from equations (7,8,9) a value of  $\gamma$  very different

from 1 can be achieved by small  $U(2)$  violating terms proportional to  $\delta a$ ,  $\delta\mu$  in the free energy.

The Landau-Ginzburg equations of motion following from the free energy (1) are:

$$\frac{\hbar^2}{2m_c}(\nabla - \frac{iq}{\hbar c}\mathbf{A})^2\psi_1 = -\mu_1\psi_1 + a|\psi_1|^3 + (a - \delta a)|\psi_2|^2\psi_1, \quad (10)$$

$$\frac{\hbar^2}{2m_c}\nabla^2\psi_2 = -\mu_2\psi_2 + a|\psi_2|^3 + (a - \delta a)|\psi_1|^2\psi_2, \quad (11)$$

$$\frac{\nabla \times (\nabla \times \mathbf{A})}{4\pi} = \frac{-iq\hbar}{2m_c c}[\psi_1^*(\nabla - \frac{iq}{\hbar c}\mathbf{A})\psi_1 - h.c.] \quad (12)$$

Now let's investigate the structure of proton vortices, which exist due to the spontaneous breaking of the  $U(1)_1$  symmetry. Such vortices are characterized by the phase of the  $\psi_1$  field varying by an integer multiple of  $2\pi$  as one traverses a contour around the core of the vortex. By continuity, the field  $\psi_1$  must vanish in the center of the vortex core. Up to this point, it has been assumed that the neutron order parameter  $\psi_2$  will remain at its vacuum expectation value in the vicinity of the proton vortex. As we have already remarked, this is not the case in many similar systems. Actually, it can be shown by using the equations of motion obtained from the free energy (1) that for most potentials  $V$ , which are approximately invariant under the  $U(2)$  symmetry, it is *impossible* for the  $\psi_2$  field to remain constant when the  $\psi_1$  field varies in space. On the other hand, from the energetic point of view, given that the potential  $V$  can be written approximately as  $V \approx U(|\psi_1|^2 + |\psi_2|^2)$ , one can argue (in part based on previous work [5, 6, 7, 8, 9, 10, 11]) that it is favorable for the  $\psi_2$  field to increase its magnitude in the vortex core to compensate the decrease in the magnitude of  $\psi_1$ .

So, anticipating a non-trivial behavior of the neutron field  $\psi_2$ , let's adopt the following cylindrically symmetric ansatz for the fields describing a proton vortex with a unit winding number:

$$\begin{aligned} \psi_1 &= \sqrt{n_1} f(r) e^{i\theta}, \\ \psi_2 &= \sqrt{n_2} g(r), \\ \mathbf{A} &= \frac{\hbar c a(r)}{q} \hat{\theta} \end{aligned} \quad (13)$$

where  $(r, \theta)$  are the standard polar coordinates. Here we assume that the proton vortex is sufficiently far from any rotational neutron vortices, so that any variation of  $\psi_2$  is solely due to the proton vortex. The functions  $f$ ,  $g$ , and  $a$  obey the following boundary conditions:  $f(0) = 0$ ,  $f(\infty) = 1$ ,  $g'(0) = 0$ ,  $g(\infty) = 1$ ,  $a(0) = 0$ , and  $a(\infty) = 1$ . We see that the fields  $\psi_1$  and  $\psi_2$  approach their vacuum expectation values at  $r = \infty$ .

The London penetration depth  $\lambda$  and the coherence length  $\xi$  of the proton superconductor will be introduced

in the standard fashion:

$$\lambda = \sqrt{\frac{m_c c^2}{4\pi q^2 n_1}}, \quad (14)$$

$$\xi = \sqrt{\frac{\hbar^2}{2m_c n_1 a}}. \quad (15)$$

We wish to find the asymptotic behavior of fields  $\psi_1$ ,  $\psi_2$  and  $\mathbf{A}$  far from the proton vortex core, as this will determine whether distant vortices repel or attract each other. The asymptotic behavior can be found analytically by expanding the fields defined in (13):

$$\begin{aligned} f(r) &= 1 + F(r), \\ g(r) &= 1 + G(r), \\ a(r) &= 1 - rS(r), \end{aligned} \quad (16)$$

so that far away from the vortex core,  $F, G, rS \ll 1$  and  $F, G, S \rightarrow 0$  as  $r \rightarrow \infty$ . This allows us to linearize far from the vortex core the equations of motion (10,11,12) corresponding to the free energy (1) to obtain:

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\begin{pmatrix} F \\ G \end{pmatrix} = \mathbf{M}\begin{pmatrix} F \\ G \end{pmatrix}, \quad (17)$$

$$S'' + \frac{1}{r}S' - \frac{1}{r^2}S = \frac{1}{\lambda^2}S, \quad (18)$$

where all derivatives are with respect to  $r$  and the matrix  $\mathbf{M}$  mixing the fields  $F$  and  $G$  is,

$$\mathbf{M} = \frac{4m_c}{\hbar^2} \begin{pmatrix} a & a - \delta a \\ a - \delta a & a \end{pmatrix} \begin{pmatrix} n_1 & 0 \\ 0 & n_2 \end{pmatrix} \quad (19)$$

Here we assume that  $(rS)^2 \ll F, G$ , i.e. the superconductor is not in the strong type-II regime (this is justified since we are only attempting to find the boundary between type-I and type-II superconductivity). The solution to Eq. (18) is known to be:

$$S = \frac{C_A}{\lambda} K_1(r/\lambda) \quad (20)$$

where  $K_1$  is the modified Bessel function and  $C_A$  is an arbitrary constant. The remaining equation (17) can be solved by diagonalizing the mixing matrix  $\mathbf{M}$ . In previous works the influence of the neutron condensate on the proton vortex was neglected, which formally amounts to setting the off-diagonal term  $M_{12}$  in Eq. (19) to 0. In that case, one can assume that the neutron field remains at its vacuum expectation value, i.e.  $G = 0$ , to obtain,

$$F = C_F K_0(\sqrt{2}r/\xi) \quad (21)$$

where  $K_0$  is the modified Bessel function. It is estimated that  $\lambda \sim 80$  fm and  $\xi \sim 30$  fm [1], which leads to  $\kappa = \lambda/\xi \sim 3$  for the Landau-Ginzburg parameter. As is known from conventional superconductors, if  $\kappa > 1/\sqrt{2}$ ,

distant vortices repel each other leading to type-II behavior. This is the standard picture of the proton superconductor in neutron stars that is widely accepted in the astrophysics community.

However, the standard procedure described above is inherently flawed since the system exhibits an approximate  $U(2)$  symmetry, and therefore the couplings  $a_{ij}$  are approximately equal  $a_{11} \approx a_{22} \approx a_{12}$ . This makes the mixing matrix  $\mathbf{M}$  nearly degenerate. The general solution to Eq. (17) is:

$$\begin{pmatrix} F \\ G \end{pmatrix} = \sum_{i=1,2} C_i K_0(\sqrt{\nu_i} r) \mathbf{v}_i \quad (22)$$

where  $\nu_i$  and  $\mathbf{v}_i$  are the eigenvalues and eigenvectors of matrix  $\mathbf{M}$ , and  $C_i$  are constants to be calculated by matching to the solution of the original nonlinear equations of motion. We would like to introduce the parameter  $\epsilon$  (which measures the asymmetry between the proton and neutron Cooper pairs) defined in the following way:

$$\epsilon = (a_{11}a_{22} - a_{12}^2)/a_{ij}^2 \simeq 2\frac{\delta a}{a}. \quad (23)$$

We should remark here that our qualitative results do not depend on the value of  $\gamma \equiv n_1/n_2$ . Indeed, no matter what  $n_1$  and  $n_2$  are, the mixing matrix is still singular in the limit  $\epsilon \rightarrow 0$ . Hence, we still get one eigenvalue which vanishes when  $\epsilon \rightarrow 0$ . So, the only crucial assumption is  $\epsilon \ll 1$ . However, to simplify our formula for the eigenvalues in what follows we assume a specific value of  $\gamma \equiv n_1/n_2 \ll 1$ . It simply allows our results to be expressed in a more transparent way. In the limit  $\gamma = n_1/n_2 \ll 1$  and  $\epsilon = 2\delta a/a \ll 1$  one can estimate the eigenvalues and eigenvectors of the matrix  $\mathbf{M}$  as:

$$\nu_1 \simeq \frac{2\epsilon}{\xi^2}, \quad \mathbf{v}_1 \simeq \begin{pmatrix} -1 \\ \gamma \end{pmatrix}, \quad (24)$$

$$\nu_2 \simeq \frac{2}{\gamma\xi^2}, \quad \mathbf{v}_2 \simeq \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \quad (25)$$

The physical meaning of solution (22) is simple: there are two modes in our two component system. The first mode describes fluctuations of relative density (concentration) of two components and the second mode describes fluctuations of overall density of two components. Notice that  $\nu_1 \ll \nu_2$ , and hence the overall density mode has a much smaller correlation length than the concentration mode. Therefore, far from the vortex core, the contribution of the overall density mode can be neglected, and one can write:

$$\begin{pmatrix} F \\ G \end{pmatrix} (r \rightarrow \infty) \simeq C_1 K_0(\sqrt{2\epsilon}r/\xi) \cdot \begin{pmatrix} -1 \\ \gamma \end{pmatrix} \quad (26)$$

The most important result of the above discussion is that the distance scale over which the proton and neutron condensates tend to their vacuum expectation values near a

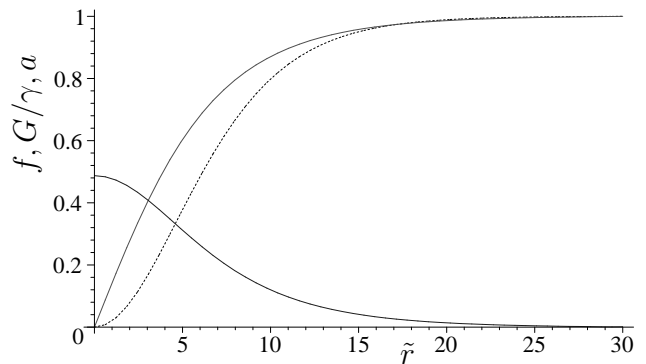


FIG. 1: In this figure we show the functions  $f(\tilde{r})$ ,  $G(\tilde{r})/\gamma$ , and  $a(\tilde{r})$  (defined in Eqs. (13,16)) as a function of the dimensionless radial coordinate  $\tilde{r} = r/\xi$ . The dotted line corresponds to  $a(\tilde{r})$ , the solid line approaching 1 at large  $\tilde{r}$  corresponds to  $f(\tilde{r})$ , and the solid line approaching 0 at large  $\tilde{r}$  corresponds to  $G(\tilde{r})/\gamma$ .

proton vortex is of order  $\xi/\sqrt{\epsilon}$  - the correlation length of the concentration mode. Since  $\epsilon \ll 1$ , this distance scale can be much larger than the proton correlation length  $\xi$ , which is typically assumed to be the radius of the proton vortex core. The appearance of the concentration mode is not surprising since we presented some arguments (before calculations) supporting the picture that the neutron condensate will increase its magnitude slightly in the vortex core, while the proton condensate will decrease its magnitude to 0 in the core center. We note that in the limit  $\epsilon \rightarrow 0$  the size of the proton vortex core becomes infinitely large, and the vortex is thereby destroyed. This is in accordance with the topological arguments, which state that if the  $U(2)$  symmetry were exact with  $\epsilon \equiv 0$ , and it is spontaneously broken to  $U(1)$ , there will be no stable vortices in the system.

We have also verified the above results numerically by solving the equations of motions (10,11,12) with a particular choice of the approximately  $U(2)$  symmetric interaction potential  $V$ . Our numerical results support the analytical calculations given above. Namely, we find that the magnitude of the neutron condensate is slightly increased in the vortex core, the radius of the magnetic flux tube is of order  $\lambda$  and the radius of the proton vortex core is of order  $\xi/\sqrt{\epsilon}$ . In Fig. 1 we show the numerical solution of the profiles of the proton vortex ( $f(\tilde{r})$ ), neutron condensate ( $G(\tilde{r})/\gamma$ ), and  $a(\tilde{r})$  (related to the gauge field through Eq. (16)) as a function of the dimensionless radial coordinate  $\tilde{r} = r/\xi$ , where  $\xi$  is the coherence length (14). We have used  $\kappa = 3$ ,  $n_1/n_2 = 0.05$ , and  $\epsilon = 0.02$  in this numerical solution.

### III. VORTEX-VORTEX INTERACTION

Now that we know the approximate solution for the proton vortex, we will proceed to look at the interac-

tion between two widely separated proton vortices. If the interaction between two vortices is repulsive, it is energetically favorable for the superconductor to organize an Abrikosov vortex lattice with each vortex carrying a single magnetic flux quantum. As the magnetic field is increased, more vortices will appear in the material. This is classic type-II behavior. If the interaction between two vortices is attractive, it is energetically favorable for  $n$  vortices to coalesce and form a vortex of winding number  $n$ . This is type-I behavior. Typically, the Landau-Ginzburg parameter  $\kappa = \lambda/\xi$  is introduced. In a conventional superconductor, if  $\kappa < 1/\sqrt{2}$  then the superconductor is type-I and vortices attract. If  $\kappa > 1/\sqrt{2}$  then vortices repel each other and the superconductor is type-II. As mentioned above, the typical value for a neutron star is  $\kappa \sim 3$ , so one could naively expect that the proton superfluid is a type-II superconductor.

The case we considered in the previous section has one new element,  $\epsilon$  which was not present in the standard type-I/II classification. However, we expect that analogous classification should remain in effect for the proton vortices described above. In such an analysis the coherence length  $\xi$  should be replaced by the actual size of the proton vortices  $\delta \sim \xi/\sqrt{\epsilon}$ . Therefore, we will define a new Landau-Ginzburg parameter for our case,

$$\kappa_{np} = \frac{\lambda}{\delta} = \sqrt{\epsilon} \frac{\lambda}{\xi}. \quad (27)$$

We expect type-I behavior with vortices attracting each other if  $\kappa_{np} \ll 1$  and type-II behavior if  $\kappa_{np} \gg 1$ . For relatively small  $\epsilon$  such an argument would immediately suggest that for the typical parameters of the neutron stars type-I superconductivity is realized (rather than the naively assumed type-II superconductivity). In what follows we present several different arguments supporting this claim.

### A. Calculating in the intervortex potential: method I

In order to make this qualitative discussion more concrete, we will present three different calculations supporting our claim that for the typical parameters of a neutron star the proton superconductor may be type-I rather than type-II. First of all, we follow the method suggested originally in [13] to calculate the force between two widely separated vortices. The methods of [13] were subsequently applied in [14] to the case similar to ours, the interaction of two widely separated vortices that have nontrivial core structure. In these papers, the force between two widely separated vortices is calculated by using a linearized theory with point sources added at the location of the vortices. The point sources in the linearized theory are chosen to produce fields matching the long distance asymptotics of the original theory. Therefore, we expand the free energy (1) up to quadratic order in the fields  $F$ ,  $G$ , and  $\mathbf{A}$  introduced in the previous section

eliminating the phase of  $\psi_1$  field in favor of the longitudinal component of  $\mathbf{A}$ , to produce a non-interacting free energy  $\mathcal{F}_{free}$ .

$$\begin{aligned} \mathcal{F}_{free} = & \int d^2r \left[ \frac{\hbar^2}{2m_c} (n_1(\nabla F)^2 + n_2(\nabla G)^2) \right. \\ & + \frac{1}{8\pi} ((\nabla \times \mathbf{A})^2 + \frac{1}{\lambda^2} \mathbf{A}^2) + 2an_1^2 F^2 \\ & \left. + 4(a - \delta a)n_1n_2 FG + 2an_2^2 G^2 \right] \end{aligned} \quad (28)$$

Following [13, 14], we must also add the source terms for each field to model the vortices:

$$\mathcal{F}_{source} = \int d^2r (\rho F + \tau G + \mathbf{j} \cdot \mathbf{A}). \quad (29)$$

The solutions to the equations of motion following from  $\mathcal{F}_{free}$  coupled to the sources can be obtained in the same manner as was done in [14]. The equations of motion resulting from  $\mathcal{F}_{free} + \mathcal{F}_{source}$  are:

$$(\nabla^2 - \mathbf{M}) \begin{pmatrix} F \\ G \end{pmatrix} = \frac{m_c}{\hbar^2} \begin{pmatrix} \rho/n_1 \\ \tau/n_2 \end{pmatrix}, \quad (30)$$

$$\nabla^2 \mathbf{A} - \frac{1}{\lambda^2} \mathbf{A} = 4\pi \mathbf{j} \quad (31)$$

where  $\mathbf{M}$  is the same mixing matrix given in Eq. (19). Since we are interested in the asymptotic behavior, we will choose the the first eigenvalue/eigenvector solution (24) that diagonalized  $\mathbf{M}$ . Therefore, the asymptotic field solutions are given by Eqs. (20,26):

$$F \simeq -G/\gamma \simeq -C_1 K_0(\sqrt{2\epsilon}r/\xi), \quad (32)$$

$$\mathbf{A} \simeq \frac{\hbar c}{q\lambda} C_A K_1(r/\lambda) \hat{\theta}.$$

Following [14], we require that (32) are solution of the equations of motion (30) and (31). The source solutions can immediately be written down when we compare Eqs. (30) and (31) with the following Bessel equations:

$$(\nabla^2 - \mu^2) K_0(\mu x) = -2\pi \delta(\mathbf{x}) \quad (33)$$

$$(\nabla^2 - \mu^2) \frac{x_j}{x} K_1(\mu x) = \frac{2\pi}{\mu} \partial_j \delta(\mathbf{x}) \quad (34)$$

The source solutions that solve the equations of motion along with (32) are:

$$\rho \simeq -\tau \simeq \frac{1}{2} \left( \frac{\hbar c}{q\lambda} \right)^2 C_1 \delta^2(\mathbf{r}), \quad (35)$$

$$\mathbf{j} \simeq -\frac{\hbar c}{2q} C_A \nabla \times (\delta^2(\mathbf{r}) \hat{z})$$

Since the equations of motion corresponding to  $\mathcal{F}_{free}$  coupled to  $\mathcal{F}_{source}$  are linear, the two-vortex solution can be considered as the sum of two single vortices at positions  $\mathbf{r}_1$  and  $\mathbf{r}_2$ . To calculate the vortex-vortex interaction energy, we use ansatz  $(F, G, \mathbf{A}) = (F_1 + F_2, G_1 +$

$G_2, \mathbf{A}_1 + \mathbf{A}_2$ )  $(\rho, \tau, \mathbf{j}) = (\rho_1 + \rho_2, \tau_1 + \tau_2, \mathbf{j}_1 + \mathbf{j}_2)$  in  $\mathcal{F}_{free} + \mathcal{F}_{source}$  and subtract off the energy of each isolated vortex. The notation 1,2 indicates that these are functions of  $\mathbf{r} - \mathbf{r}_{1,2}$ . Using the equations of motion (30,31) the interaction energy can be written as

$$\mathcal{F}_{int} = \int d^2r (\mathbf{j}_1 \cdot \mathbf{A}_2 + \rho_1 F_2 + \tau_1 G_2) \quad (36)$$

Substituting the asymptotic field solutions (32,35) into the interaction energy given above, the integration can be done as in [14] to obtain the following expression for the interaction energy per unit vortex length of two widely separated parallel vortices:

$$U(d) \simeq \frac{2\pi\hbar^2 n_1}{m_c} (C_A^2 K_0(d/\lambda) - C_1^2 (1 + \mathcal{O}(\gamma)) K_0(\sqrt{2}\epsilon d/\xi)) \quad (37)$$

where  $d = |\mathbf{r}_1 - \mathbf{r}_2| \rightarrow \infty$  is the separation between the two vortices. We see that if the first term in  $U$  dominates as  $d \rightarrow \infty$  then the potential is repulsive, otherwise, if the second term dominates the potential is attractive. In other words, if  $\sqrt{\epsilon}\lambda/\xi < 1/\sqrt{2}$ , then vortices attract each other and the superconductor is type-I; otherwise, vortices repel each other and the superconductor is type-II. This confirms our original qualitative argument that  $\kappa_{np} = \lambda/\delta = \sqrt{\epsilon}\lambda/\xi$  should be considered as an effective Landau-Ginzburg parameter, which determines the boundary between the type-I and type-II proton superconductivity. In terms of the parameters of our theory, we have

$$\kappa_{np} = \frac{\lambda}{\delta} = \sqrt{\epsilon} \frac{\lambda}{\xi} = \frac{m_c c}{\sqrt{\pi} \hbar} \frac{\sqrt{\delta a}}{q}. \quad (38)$$

In this case we see that the type-I/II behavior is controlled in part by the degree of symmetry breaking (proportional to  $\delta a$ ). Our numerical estimates (see conclusion) suggest that  $\kappa_{np} < 1/\sqrt{2}$ , and therefore, the system is a type-I superconductor.

## B. Calculating the intervortex potential: method II

Due to the importance and far reaching consequences of this result, we have also calculated the vortex-vortex interaction energy in a more direct way following [15], where the introduction of the auxiliary sources is completely avoided. This method, very different in nature, has produced the same result (37) as the above procedure, therefore confirming our picture.

We proceed as follows. First, we again eliminate the phase of  $\psi_1$  with an appropriate gauge transformation. Now, recalling our definitions (13, 16), let  $(F, G, \mathbf{A}) = (F_1, G_1, \mathbf{A}_1)$  be the exact fields of a single vortex located at  $\mathbf{r}_1$ . Also let  $(F, G, \mathbf{A}) = (F_1 + F_2, G_1 + G_2, \mathbf{A}_1 + \mathbf{A}_2)$  be the exact fields produced by two vortices at locations  $\mathbf{r}_1$  and  $\mathbf{r}_2$ . Note that subscript 2 does not generally refer here to functions of  $\mathbf{r} - \mathbf{r}_2$  as it did in the previous subsection: here  $(F_2, G_2, \mathbf{A}_2)$  are just corrections to the fields of the single vortex at  $\mathbf{r}_1$ . When  $\mathbf{r}$  is far from the core of vortex 1,  $F_1, G_1, \mathbf{A}_1$  are small and the equations of motion can be linearized to yield familiar asymptotics (32). Moreover, when  $\mathbf{r}$  is far from the cores of both vortices,  $F_i, G_i, \mathbf{A}_i$  are small for both  $i = 1, 2$  and we obtain:

$$F_i \simeq -G_i/\gamma \simeq -C_1 K_0(\sqrt{2}\epsilon|\mathbf{r} - \mathbf{r}_i|/\xi), \quad (39)$$

$$\mathbf{A}_i \simeq \frac{\hbar c}{q\lambda} C_A K_1(|\mathbf{r} - \mathbf{r}_i|/\lambda) \hat{\theta}.$$

To calculate the interaction energy of the two vortices, let us divide the space into two cells  $T_1$  and  $T_2$ , which contain the centers of vortices 1 and 2 respectively (if we had more than two vortices in our problem these would be the Wigner-Seitz cells of the vortex lattice). For simplicity, we will take the boundary between two cells to be the line perpendicular to and passing through the middle of the line-segment joining the cores of two vortices. This boundary then acts as an axis of symmetry for the problem: the total energy of two interacting vortices is just twice the energy of each cell. The vortex-vortex interaction energy is then:

$$\begin{aligned} \mathcal{F}_{int} &= \mathcal{F}[F_1 + F_2, G_1 + G_2, \mathbf{A}_1 + \mathbf{A}_2] - 2\mathcal{F}[F_1, G_1, \mathbf{A}_1] \\ &= 2 \int_{T_1} d^2r (\mathcal{E}[F_1 + F_2, G_1 + G_2, \mathbf{A}_1 + \mathbf{A}_2] \\ &\quad - \mathcal{E}[F_1, G_1, \mathbf{A}_1]) - 2 \int_{T_2} d^2r \mathcal{E}[F_1, G_1, \mathbf{A}_1] \end{aligned} \quad (40)$$

where  $\mathcal{E} = d\mathcal{F}/dr^2$  is the free energy per unit volume and subscripts  $T_i$  indicate integration over the corresponding cells. We assume the vortex separation to be large, therefore the fields  $F_1, G_1, \mathbf{A}_1$  inside cell  $T_2$  will be small, so we can expand the integral over the cell  $T_2$  in (40) to second order in  $F_1, G_1, \mathbf{A}_1$ . Similarly, the corrections to fields  $F_1, G_1, \mathbf{A}_1$  inside cell  $T_1$  will be small so we can expand the integral over the cell  $T_1$  in (40) to second order in corrections  $F_2, G_2, \mathbf{A}_2$ . The resulting expression can be simplified by using equations of motion (10), (11), (12) to obtain:

$$\begin{aligned}\mathcal{F}_{int} \simeq & 2 \oint_{T_1} \mathbf{dS} \cdot \left( \frac{\hbar^2 n_1}{2m_c} F_2 (2\nabla F_1 + \nabla F_2) + \frac{\hbar^2 n_2}{2m_c} G_2 (2\nabla G_1 + \nabla G_2) + \frac{1}{8\pi} \mathbf{A}_2 \times (2\nabla \times \mathbf{A}_1 + \nabla \times \mathbf{A}_2) \right) \\ & - 2 \oint_{T_2} \mathbf{dS} \cdot \left( \frac{\hbar^2 n_1}{2m_c} F_1 \nabla F_1 + \frac{\hbar^2 n_2}{2m_c} G_1 \nabla G_1 + \frac{1}{8\pi} \mathbf{A}_1 \times (\nabla \times \mathbf{A}_1) \right)\end{aligned}\quad (41)$$

Here the integrals are over the boundary of cells  $T_1$  and  $T_2$ . Since this boundary is far away from either vortex center, we can use the asymptotic expressions (39) for the fields  $(F_i, G_i, \mathbf{A}_i)$  to explicitly calculate the surface integrals in (41). We note that by symmetry of the asymptotic solution on the boundary, the second integral in (41) cancels with the part of the first integral to yield:

$$\begin{aligned}\mathcal{F}_{int} \simeq & 2 \oint_{T_1} \mathbf{dS} \cdot \left( \frac{\hbar^2 n_1}{m_c} F_2 \nabla F_1 + \frac{\hbar^2 n_2}{m_c} G_2 \nabla G_1 \right. \\ & \left. + \frac{1}{4\pi} \mathbf{A}_2 \times (\nabla \times \mathbf{A}_1) \right)\end{aligned}\quad (42)$$

Substituting asymptotic solutions (39) into the above, we find the vortex-vortex interaction energy per unit length to be:

$$\begin{aligned}U(d) = & \frac{2\pi\hbar^2 n_1}{m_c} (C_A^2 K_0(d/\lambda) \\ & - C_1^2 (1 + \mathcal{O}(\gamma)) K_0(\sqrt{2\epsilon} d/\xi))\end{aligned}\quad (43)$$

in accordance with our previous result (37).

#### IV. CRITICAL MAGNETIC FIELDS

Our third check of the main result that for relatively small  $\epsilon$  the superconductor in the neutron stars may be, in fact, type-I is based on the calculation of the critical magnetic fields. Usually one calculates the critical magnetic fields  $H_c$  and  $H_{c2}$ . These are the physically meaningful fields above which the superconductivity is destroyed in type-I and type-II superconductors respectively. If  $H_c > H_{c2}$  then the superconductor is type-I, otherwise, the superconductor is type-II.

First, we will calculate the critical magnetic field  $H_c$ . This is defined as the point at which the Gibbs free energy of the normal phase is equal to the Gibbs free energy of the superconducting phase. In other words, as the external magnetic field  $H$  is increased above  $H_c$ , it is energetically favorable for the superconducting state to be destroyed macroscopically. The Gibbs free energy in the presence of an external magnetic field  $H$  is:

$$g(H, T) = f(B, T) - \frac{BH}{4\pi} \quad (44)$$

where  $H$  is the external magnetic field,  $B$  is the magnetic induction, and  $T$  denotes temperature. The quantity  $f(B, T)$  is the integrand of the free energy density (given

by Eq. (1)) over a superconducting sample. For the superconducting state where  $\langle |\psi_1|^2 \rangle = n_1$ ,  $\langle |\psi_2|^2 \rangle = n_2$ , and  $B = 0$  (Meissner effect), the Gibbs free energy is

$$g_s(H, T) = -\frac{\mu^2}{2a} - \frac{(\delta\mu)^2}{\delta a} - \delta a \left( \frac{\mu}{2a} \right)^2, \quad (45)$$

where we expressed the result in terms of the parameters of Eq. (1). We also replaced the densities  $(n_1 + n_2) \rightarrow \mu/a$  and  $(n_2 - n_1) \rightarrow 2\delta\mu/\delta a$  in terms of the same parameters by neglecting small factors  $\sim \delta a/a$ . For the normal state, we have  $\langle |\psi_1|^2 \rangle = 0$ ,  $\langle |\psi_2|^2 \rangle = (\mu + \delta\mu)/a$ , and  $B = H$ . The Gibbs free energy is:

$$g_n(H, T) = -\frac{H^2}{8\pi} - \frac{\mu^2}{2a} - \frac{\mu\delta\mu}{a} \quad (46)$$

As  $H_c$  is defined as the point at which  $g_s(H_c) = g_n(H_c)$ , we can solve for the critical field. Equating the free energies of the normal and superconducting state, we find

$$H_c = \sqrt{8\pi\delta a} \left( \frac{\mu}{2a} - \frac{\delta\mu}{\delta a} \right) \rightarrow n_1 \sqrt{8\pi\delta a}, \quad (47)$$

where at the final stage we used the equation for  $n_1$  in the superconducting phase expressed in terms of the original parameters (3).

Now we will proceed to calculate  $H_{c2}$ . This is the critical magnetic field below which it becomes energetically favorable for a microscopic region of the superconducting state to be nucleated, with the normal state occurring everywhere else in space. In order to calculate  $H_{c2}$ , we follow the standard procedure and linearize the equations of motion for  $\psi_1$  about the normal state with  $\langle |\psi_1|^2 \rangle = 0$  and  $\langle |\psi_2|^2 \rangle = (\mu + \delta\mu)/a$ . The linearized equation of motion reads,

$$\frac{\hbar^2}{2m_c} \left( -i\nabla - \frac{q}{\hbar c} \mathbf{A} \right)^2 \psi_1 = \omega \psi_1, \quad (48)$$

$$\omega = (\mu + \delta\mu) \frac{\delta a}{a} - 2\delta\mu. \quad (49)$$

This is simply a Schrodinger equation for a particle in a magnetic field, with an energy of  $\omega$ . This is a standard quantum mechanics problem and we can immediately write down the solution. The first Landau level is the ground state energy of  $\epsilon_0(H) = \hbar|q|H/2m_c c$ . Therefore, if  $\omega < \epsilon_0$ , then only the trivial solution with  $\psi_1 = 0$  is possible. The critical field  $H_{c2}$  is defined as the point at which  $\omega = \epsilon_0(H_{c2})$ . This is given as

$$H_{c2} = \frac{2m_c c}{\hbar|q|} \left[ (\mu + \delta\mu) \frac{\delta a}{a} - 2\delta\mu \right] \simeq \frac{4m_c c}{\hbar|q|} \delta a n_1 \quad (50)$$



Now that we have the critical fields  $H_c$  and  $H_{c2}$  in hand, we can compare the two in order to determine the type-I/II nature. If  $H_c < H_{c2}$  this means that it is energetically favorable for microscopic regions of the superconducting state to be nucleated as  $H$  is decreased. This is type-II behavior, and this nucleation manifests itself in the form of an vortex lattice. If  $H_c > H_{c2}$ , then it is energetically favorable for macroscopic regions of the superconducting state to be present as  $H$  is decreased. This is a type-I superconductor and the superconducting state persists everywhere in the material when  $H < H_c$ , as opposed to a type-II superconductor where it is localized in regions of space in between the vortices. From Eqs. (3,47,49,50) we see that

$$\frac{H_{c2}}{H_c} \simeq \sqrt{2} \frac{m_c c}{\sqrt{\pi} \hbar} \frac{\sqrt{\delta a}}{q} = \sqrt{2} \kappa_{np}. \quad (51)$$

This agrees with the parametrical behavior given in Eq. (38) obtained from the vortex interaction calculation of the previous section. To estimate  $H_c$  numerically, it is convenient to represent  $H_c$  as

$$H_c = \frac{\varphi_0}{2\pi\lambda\xi} \sqrt{\frac{\delta a}{a}}, \quad \varphi_0 = \frac{2\pi\hbar c}{q} = 2 \times 10^7 \text{G} \cdot \text{cm}^2, \quad (52)$$

where  $\varphi_0$  is the quantum of the fundamental flux. If we substitute  $\lambda = 80 \text{ fm}$  and  $\xi = 30 \text{ fm}$  (typical values) in the expression for the critical magnetic field (47),  $H_c$  is estimated to be the  $H_c \simeq 10^{14} \text{ G}$ , which is smaller than the “naive” estimate by a factor of  $\sqrt{\delta a/a} \sim 10^{-1}$ . It is quite amazing that very different calculations of the critical magnetic fields (51) lead exactly to the same conclusion which was derived from analysis of the vortex-vortex interaction (38).

## V. CONCLUSION

In this paper we have demonstrated using various calculations that the proton superfluid present inside a neutron star may in fact be a type-I superconductor [4]. This supports the observation made by Link [1] that the conventional picture of type-II superconductivity may be inconsistent with the observations of long period precession in isolated pulsars [2]. The most important consequence of this paper is that whether the proton superconductor is type-I or type-II depends strongly on the magnitude of the  $SU(2)$  asymmetry parameter  $\epsilon$ . Specifically, we find that the superconductor is type-I when  $\kappa_{np} = \sqrt{\epsilon}\lambda/\xi < 1/\sqrt{2}$ , and type-II otherwise. This result is quite generic, and not very sensitive to the specific details of the interaction potential  $V$ . In particular, when  $\epsilon \rightarrow 0$  the superconductor is type-I. The parameter  $\epsilon$  is not known precisely; the corresponding microscopical calculation would require the analysis of the scattering lengths of Cooper pairs for different species. We can roughly estimate this parameter as being related to the original  $SU(2)$  isospin symmetry breaking  $\epsilon \sim (m_n - m_p)/m_n \sim 10^{-2}$ . If we

assume a typical value for  $\lambda/\xi \sim 3$  and  $\epsilon \sim 10^{-2}$ , we estimate  $\kappa_{np} = \sqrt{\epsilon}\lambda/\xi \sim 0.3 < 1/\sqrt{2}$ , which corresponds to a type-I superconductor. From these crude estimates, we see that it is very likely that neutron stars are type-I superconductors with the superconducting region devoid of any magnetic flux, as was originally suggested in [1] to resolve the inconsistency with observations of long period precession [2] in isolated pulsars. Many mechanisms have been proposed to explain glitches [17]. If the proton superfluid exhibits type-I superconductivity then some explanations of glitches [18] that assume type-II superconductivity would have to be reconsidered, as suggested in Ref. [1]. It might be interesting to consider how the presence of a nonzero proton condensate affects the characteristics of the neutron vortices that carry angular momentum. In particular, as we have demonstrated for magnetic flux tubes in this paper, the neutron vortices might have an enhanced proton superfluid density inside, as well as a coherence length  $\xi_n$  (the approximate size of the vortex) which is much larger than originally expected. In this case, the pinning force that is related to the size of the vortex core [19] could be very different from the simple estimates when the strong interaction between the neutron and proton Cooper pairs is ignored.

If the core is indeed a type-I superconductor, the magnetic field could exist in macroscopically large regions where there are alternating domains of superconducting (type-I) matter and normal matter. In this case, neutron stars could have long period precession. Such a structure follows from few different arguments. First of all, as has been estimated [20], it takes a very long time to expel a typical magnetic flux from the neutron star core. Therefore, if the magnetic field existed before the neutron star became a type-I superconductor, it is likely that magnetic field will remain there. Another argument suggesting the same outcome follows from the fact that topology (magnetic helicity) is frozen in the environment with high conductivity; therefore, the magnetic field must remain in the bulk of the neutron star. The last argument supporting the same picture is due to Landau [21] who argued that if a body of arbitrary shape (being a type-I superconductor) is placed under influence of the external magnetic field with strength  $H < H_c$ , then the magnetic field in some parts of the body may reach the critical value  $H_c$ , while in other parts of the body it may remain smaller than the critical value,  $H < H_c$ . In this case, a domain structure will be formed, similar to ferromagnetic systems. Specifically, on a macroscopic distance scale, the magnetic flux must be embedded in the superconductor. This would mean that the superconductor is in an intermediate state as opposed to the vortex state of the type-II superconductor, which was assumed to be realized up to this point. The intermediate state is characterized by alternating domains of superconducting and normal matter. The superconducting domains will then exhibit the Meissner effect, while the normal domains will carry the required magnetic flux. The pattern of these domains is usually strongly related to the geometry of

the problem. The simplest geometry, originally considered by Landau [21], is a laminar structure of alternating superconducting and normal layers. However, it has been also argued [22] that due to the geometry of the neutron vortex lattice, the normal proton domains will be in the shape of cylinders concentric with the rotational neutron vortices.

While some precise calculations are required for understanding of the magnetic structure in this case, one can give some simple estimation of the size of the domains using the calculations Landau presented for a different geometry. His formula [21] suggests that the typical size of a domain is

$$a \sim 10\sqrt{R\Delta}, \quad (53)$$

where  $R$  is a typical external size identified with a neutron star core ( $R \sim 10$  km), while  $\Delta$  is a typical width of the domain wall separating normal and superconducting states. We estimate  $\Delta \sim \delta = \xi/\sqrt{\epsilon}$  as the largest microscopical scale of the problem. Numerically,  $a \sim 10^{-1}$  cm which implies that a typical domain with size of order  $\sim 10^{-1}$  cm can accommodate  $\sim 10$  neutron vortices separated by a distance  $\sim 10^{-2}$  cm.

The consequences of this picture still remain to be explored. In particular, it might be of importance in the explanation of glitches. It may be also important in analysis of the cooling properties of the neutron stars.

It would be very interesting to test the ideas outlined in this paper by doing laboratory experiment in the spirit of the Cosmology in the Laboratory (COSLAB) program. In particular, it would be interesting to find a condensed matter system (high  $T_c$  superconductor?) where the very interesting feature discussed in this paper can be tested. Namely, the core of the vortices and their interactions could be very different from the simple estimates. This might happen if there is a condensate of another field with energy scales almost degenerate with energies determined by the Landau-Ginsburg complex  $\psi$  field describing a superconductor. Over the last few years several experiments have been done to test ideas drawn from cosmology and astrophysics (see Ref. [7] and the web page [16] of the latest COSLAB meeting for further details).

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